Methods of Testing Lepton Universality

using Charged Current Weak Interactions

L.Stumpf
University of Victoria

July 7, 1999

1 Introduction

The fundamental particles of matter and the interactions between them are described very successfully by the Standard Model of Particle Physics [1]. However, there are many aspects of the Standard Model which at present cannot be explained. Tests of the Standard Model are continually being conducted, and searches for new physics beyond the Standard Model are increasing in precision and sensitivity. In this work, methods of testing one of the assumptions of the Standard Model, lepton universality, will be described.

The tau and the muon in the Standard Model are exact duplicates of the electron, apart from their greater mass and separately conserved lepton number. Their interactions via the fundamental forces are assumed to be identical. Each of the fundamental forces is mediated by a particle which couples to a "charge" carried by the lepton with a strength which is characteristic of the force. Under an assumption of lepton universality, the "weak charge" of the tau particle has the same magnitude as that of an electron or a muon. This implies that any interactions which arise as a consequence of the weak force have the same probability of occurrence, regardless of whether the lepton involved is a tau particle, muon, or electron. Experimental results indicating deviations from universality could be indicative of new physics.

This assumption can be tested in both the neutral-current weak sector, and the charged-current weak sector. We shall deal with only the latter. After a discussion of the Standard Model in Section 2 and decay widths in Section 3, the methods of testing lepton universality in charged-current couplings will be discussed, beginning with W decays in Section 4. Leptonic decays of tau particles and of muons are discussed in Sections 5 and 6. Results from pion decays are discussed in Section 7, and in Section 8, τ hadronic decays are compared with pion (and kaon) leptonic decays. The implications of these tests are discussed in Section 10.

2 The Standard Model

In the Standard Model, the construction and dynamics of the universe rely on two types of fundamental[1] particles, fermions (particles with half-integer spin) and bosons (particles with integer spin). The fundamental fermions are either leptons or quarks; these are the most basic constituents of matter.

[1]Fundamental particles are those with no observed substructure.

Leptons and quarks interact via the four fundamental forces: strong, weak, electromagnetic, and gravitational. At the subatomic scales studied to date, the gravitational force has no measurable effects and can be neglected. The two types of fundamental fermions can be differentiated by the ways in which they interact; quarks interact via the strong force while leptons do not, both types interact weakly, and all charged particles interact electromagnetically. The interactions involve the exchange of fundamental bosons, often called gauge bosons, which propagate the forces between the constituents of matter.

The most familiar lepton is the electron (e^-); the other charged leptons (μ^-, τ^-) apparently differ from it only in mass and in lepton number. In addition, each charged lepton has a neutral partner called a neutrino (ν_e, ν_μ, ν_τ), which only interacts via the weak force. Unlike leptons, quarks are not observed individually, but combine in twos and threes to form mesons and baryons, respectively. These are collectively known as hadrons. Familiar baryons are protons and neutrons, which are composed of u and d quarks. The quarks and leptons in the Standard Model are arranged into generations according to their mass, as shown in Table 1. The table is divided horizontally into generations; for example, the e^- and ν_e leptons together with the u and d quarks form the first generation. The fundamental bosons, shown in Table 2, are the photons, $W^\pm$ and Z^0 particles, and gluons for the electromagnetic, weak, and strong forces, respectively. In addition to the particles mentioned above, every particle has an associated antiparticle with opposite electric charge.

The Standard Model is based on a combination of three gauge groups, $SU_c(3)\times SU_L(2)\times U_Y(1)$. The form of the couplings between fermions is theoretically predicted by imposing local gauge invariance on the Standard Model; that is, by requiring that there exists a group of symmetry operations or local gauge transformations under which the measurable quantities remain invariant. The couplings between strongly interacting particles by the exchange of gluons is described by $SU_c(3)$. The other two groups, $SU_L(2) \times U_Y(1)$, together describe the unified electroweak interaction. The subscript L indicates that only the left-handed chiral component of the fermions interacts with the W bosons. The subscript Y denotes the weak hypercharge of the particle, defined by $Q = T^3 + Y/2$ where Q is the electric charge and T^3 is the third component of the weak isospin. The fermions shown in Table 1 are arranged into weak isospin doublets. For example, the ν_e and e together form a left-handed doublet, with $T^3_{\nu_e} = 1/2$ and $T^3_e = -1/2$. The same is true for the u and d quarks, respectively, and the pattern holds throughout all three

Leptons			Quarks		
Flavour	Charge	Mass(GeV)	Flavour	Charge	Mass
ν_e	0	$< 15 \times 10^{-9}$	u	$+\frac{2}{3}$	1.5 - 5 MeV
e	-1	5.11×10^{-4}	d	$-\frac{1}{3}$	3 - 9 MeV
ν_μ	0	$< 1.7 \times 10^{-4}$	c	$+\frac{2}{3}$	1.1 - 1.4 GeV
μ	-1	0.106	s	$-\frac{1}{3}$	60 - 170 MeV
ν_τ	0	$< 18 \times 10^{-3}$	t	$+\frac{2}{3}$	173.8 ± 5.2 GeV
τ	-1	1.78	b	$-\frac{1}{3}$	4.1 - 4.4 GeV

Table 1: Some properties of the fundamental constituents of matter in the Standard Model. Electric charge is given in units of electric charge. The masses given are the Particle Data Book 1998 evaluations [2].

Force	Gauge boson
Electromagnetic	photons
Strong	gluons
Weak	$W^{\pm}/Z^0$

Table 2: Exchange particles for the fundamental forces.

generations. The right-handed fermions are not part of the weak-isospin doublets, but are singlets with $T = T^3 = 0$.

In nature, the invariance of the unified electroweak group is not evident because of a process described as "spontaneous symmetry breaking". Upon symmetry breaking, the electroweak force is resolved into the electromagnetic force associated with the $U_Q(1)$ gauge symmetry, and the weak force. With the inclusion of spontaneous symmetry breaking, another boson, the Higgs particle (H^0) is introduced into the Standard Model [3], providing another channel through which the fundamental fermions may interact. Precision measurements of electroweak parameters have been used to constrain the Higgs mass through its effect in radiative corrections; the current preliminary upper limit is $M_H < 262$ GeV [2] [4]. The process $e^+e^- \rightarrow Z^0 H^0$ should be

[2]In this work, the convention $\hbar = c = 1$ will be used.

observed at LEP if the energy threshold for production of the final state particles is attained. Failure to observe this process to date at centre of mass energies of 183 GeV implies a lower limit of $M_H > 88.3$ GeV [5].

The Higgs mechanism causes some problems within the Standard Model. For example, the Higgs mass is not predicted by the model, but since the Higgs boson interacts with particles at the electroweak scale it is natural to assume that its mass will be of the same order as that scale, or around 100 GeV. However, the radiative corrections to the mass, which are calculated by first order perturbation techniques, diverge. In principle, this problem can be fixed within the Standard Model, but it requires some very heavy-handed "fine tuning" at every order in perturbation theory, which is an unsatisfactory situation [6]. In part to address this problem, the concept of supersymmetry, in which each Standard Model particle has a partner whose spin differs by 1/2, was introduced. The extra particles contribute radiative corrections that cancel the problematic divergences. In the Minimal Supersymmetric Model, the existence of four Higgs bosons (h^0, H^0, $H^{\pm}$, A^0) is postulated, which is the minimal number required by a supersymmetry theory. No direct empirical evidence of the Higgs particles, including the Standard Model Higgs, has yet been observed.

The Standard Model does not predict the magnitude of the couplings between the gauge bosons and the fermions. It is assumed, however, that the couplings of the $W^{\pm}$ and Z^0 bosons to the different generations of leptons will be identical. This assumption of *lepton universality* can be tested both in neutral weak current processes which involve a Z^0 particle, and in charged weak current processes which involve the $W^{\pm}$ particles. In this work, only the latter will be considered.

The decay of a W to each of the three lepton generations is represented by the Feynman diagrams in Figure 1. The initial state particle is shown on the left, and the final state particles are shown on the right. The *coupling constants* are given by g_l, where l is e, μ, or τ. The strength of the coupling, and hence the probability of the corresponding process, is proportional to the coupling constants. Lepton universality implies that $g_e = g_\mu = g_\tau$. In accordance with the lepton universality prediction of the Standard Model, the decay of a W should be equally likely to occur to any generation of leptons, except for phase space corrections.

Lepton universality in the charged current sector can be tested in ways other than by looking at the decay of W particles. Leptons decay via the charged weak current, that is, with a virtual W particle acting as the prop-

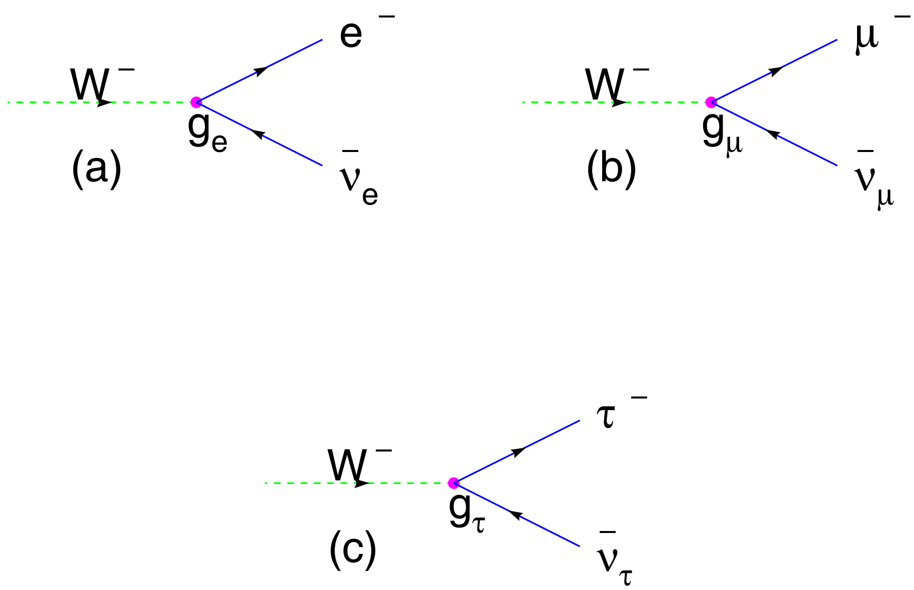

Figure 1: (a) $W \to e^-\bar{\nu}_e$ decay, (b) $W \to \mu^-\bar{\nu}_\mu$ decay, and (c) $W \to \tau^-\bar{\nu}_\tau$ decay.

agator between initial and final states. Mesons such as pions and kaons also decay via the charged weak current. These processes give us less direct but equally valid ways of testing the Standard Model. For example, examination of these processes could unveil contributions from Higgs channels, where a particle which is assumed to decay weakly through a W also decays through a charged Higgs particle. Since the Standard Model does not include a charged Higgs sector, this could be evidence of supersymmetric contributions. As the precision of the universality tests improve, they become sensitive to other aspects of the Standard Model, such as radiative corrections, the mass of the neutrino, and exotic types of couplings. If comparison with other experimental results and with theoretical models rules out factors such as those listed above, then deviations from lepton universality could imply that the τ and the μ actually differ from the electron in more than mass and lepton number.

3 Decay width and branching ratio

Lepton universality can be tested by looking at the decays of particles. The *partial decay width* of a particle is proportional to the transition rate from an initial state to a particular final state, and depends upon the coupling strength of the interaction. It is given by

$$d\Gamma = \frac{|M|^2}{2m} d\phi \tag{1}$$

where $d\phi$ is the Lorentz invariant phase space factor which corresponds to kinematic constraints such as conservation of 4-momentum, and m is the mass of the decaying particle. The matrix element, M, takes into account the dynamics of the process such as the strength of the coupling between particles.

Many particles can decay via more than one decay channel or mode. The branching ratio is the fractional number of decays via a specific mode, and is given by the ratio of the partial decay width to the total decay width. For example, a W particle may decay via the channel $W^- \to e^- \bar{\nu}_e$, with a corresponding branching ratio given by

$$B\left(W^- \to e^- \bar{\nu}_e\right) = \frac{\Gamma\left(W^- \to e^- \bar{\nu}_e\right)}{\Gamma_W}. \tag{2}$$

The total width Γ_W in the denominator of Equation 2 is the inverse of the lifetime.

4 The leptonic decays of the W

The W particle can decay to leptons or to quarks, with $B(W \to l\nu_l)$ approximately 32% for the combined lepton channels, and $B(W \to qq)$ approximately 68%. Figure 1 shows the Feynman diagrams for the decays of a real W to the three lepton generations. Assuming massless final state leptons, the following partial width is obtained:

$$\Gamma(W \to l\bar{\nu}_l) = \frac{g_l^2 m_W}{48\pi}, \tag{3}$$

where l is either e, μ, or τ, corresponding to the final state fermions, and m_W is the mass of the W particle. The effects of the fermion masses and radiative processes are below 0.4% [7], and are negligible compared with the precision of the branching ratio measurements. The partial width is related to the branching ratio as shown in Equation 2, hence g_τ/g_e and g_μ/g_e can be obtained from the ratio of the corresponding branching ratios.

The branching ratios are measured at two types of facilities, e^+e^- colliders such as LEP, and $p\bar{p}$ colliders such as the Tevatron, which currently are the only facilities operating at the centre of mass energies required to produce real W particles. At LEP, W particles are usually produced in pairs via the processes shown in Figure 2 [8]; each W subsequently decays to fermions. The three possible types of WW events are: $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$,

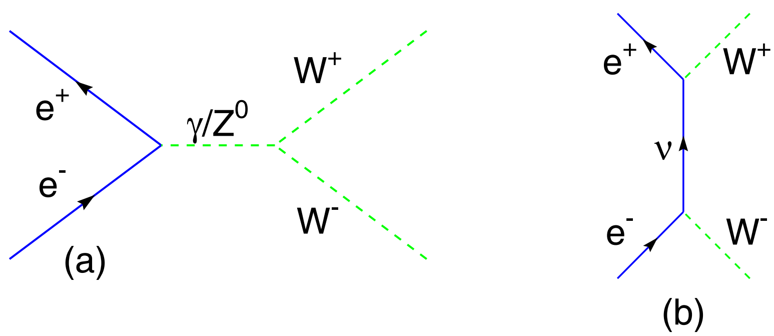

Figure 2: W production at LEP via the s-channel (a), and t-channel (b).

$e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}l\nu$ and $e^+e^- \rightarrow W^+W^- \rightarrow l\nu l\nu$. Although a generic $e^+e^- \rightarrow W^+W^-$ event is difficult to separate from the other events, it is possible to separate each of these three topologies individually by looking for the appropriate final state fermions. The number of events selected in each channel are then used in a series of simultaneous fits to determine the branching ratios B($W \rightarrow qq$), B($W \rightarrow e\nu$), B($W \rightarrow \mu\nu$), B($W \rightarrow \tau\nu$), and the total cross section for $e^+e^- \rightarrow W^+W^-$. More information on the branching ratio calculation is found in Appendix A.1.

The branching ratios were calculated at LEP at centre of mass energies of 161, 172, and 183 GeV, but the results are dominated by the 183 GeV data. Results from the OPAL experiment show 873 W^+W^- events, divided between 362 $W^+W^- \to q\bar{q}l\nu$ events, 78 $W^+W^- \to l\nu l\nu$ events, and 433 $W^+W^- \to q\bar{q}q\bar{q}$ events [9]. The other three LEP experiments show similar numbers [8], for a total of approximately 4000 W pair events. The branching ratios measured by the four LEP experiments are given in Table 3 [4]. The assumption that $\mathrm{B}(W \to e\nu_e) + \mathrm{B}(W \to \mu\nu_\mu) + \mathrm{B}(W \to \tau\nu_\tau) + \mathrm{B}(W \to q\bar{q}) = 1$ is used in the determination of the branching ratios [9].

Experiment	$\mathrm{B}(W \to e\nu_e)\%$	$\mathrm{B}(W \to \mu\nu_\mu)\%$	$\mathrm{B}(W \to \tau\nu_\tau)\%$	$\mathrm{B}(W \to q\bar{q})\%$
ALEPH	$11.2 \pm 0.8 \pm 0.3$	$9.9 \pm 0.8 \pm 0.2$	$9.7 \pm 1.0 \pm 0.3$	$69.0 \pm 1.2 \pm 0.6$
DELPHI	$9.9 \pm 1.1 \pm 0.5$	$11.4 \pm 1.1 \pm 0.5$	$11.2 \pm 1.7 \pm 0.7$	$67.5 \pm 1.5 \pm 0.9$
L3	$10.5 \pm 0.9 \pm 0.2$	$10.2 \pm 0.9 \pm 0.2$	$9.0 \pm 1.2 \pm 0.3$	$70.1 \pm 1.3 \pm 0.4$
OPAL	$11.7 \pm 0.9 \pm 0.3$	$10.1 \pm 0.8 \pm 0.3$	$10.3 \pm 1.0 \pm 0.3$	$67.9 \pm 1.2 \pm 0.6$
LEP Average	10.9 ± 0.5	10.3 ± 0.5	10.0 ± 0.6	68.8 ± 0.8

Table 3: LEP preliminary results for the W branching ratios, in percent. In each case, the first error is statistical, and the second is systematic.

Moulik [8] has used the combined LEP branching ratios to calculate the following ratios of coupling constants:

$$\frac{g_\mu}{g_e} = 0.971 \pm 0.031$$

and

$$\frac{g_\tau}{g_e} = 0.954 \pm 0.040.$$

If we use the results from each individual experiment as shown in Table 3, and form an error estimate by adding the statistical and systematic errors in quadrature and assuming no correlation between the results for the various leptonic modes of decay, we find that the ratios of the coupling constants are as given in Table 4. Of the three possible ratios, only two are independent; the third one can be calculated from the other two. However, in this work we wish to have them all available for comparison purposes with the results from other types of universality tests.

At proton-antiproton colliders, the W particles are produced singly. A typical interaction is shown in Figure 3; a quark in the proton is shown interacting with an antiquark in the antiproton, and the W is shown decaying

Experiment	g_μ/g_e	g_τ/g_e	g_τ/g_μ
ALEPH	0.94 ± 0.05	0.93 ± 0.06	0.99 ± 0.06
DELPHI	1.07 ± 0.09	1.06 ± 0.11	0.99 ± 0.10
L3	0.99 ± 0.06	0.93 ± 0.07	0.94 ± 0.08
OPAL	0.93 ± 0.05	0.94 ± 0.06	1.01 ± 0.07
D0	0.94 ± 0.05	1.00 ± 0.03	1.07 ± 0.06
CDF	1.01 ± 0.04	0.97 ± 0.07	0.96 ± 0.07
Average	0.97 ± 0.02	0.98 ± 0.02	1.00 ± 0.03

Table 4: Universality tests using W decay branching ratios from the LEP and Tevatron experiments.

leptonically. The other quarks in the proton and antiproton do not take part in the interaction, and will recombine to form other hadrons, typically pions, which are represented below with an X. The topologies of interest are $p\bar{p} \to WX$, where the W decays leptonically. As with the e^+e^- colliders, a generic $p\bar{p} \to WX$ event is difficult to separate from the other events, but it is possible to separate the topologies individually by looking for the final state leptons resulting from the decay of the W.

Unlike the situation in e^+e^- colliders, here the total cross section for the interaction will depend upon the specific constituents in the proton and antiproton which interact, and on their energies. Since this varies from one event to the next, the quantity that is measured is the total cross section multiplied by the branching ratio, $\sigma_W B(W \to e\nu_e)$, $\sigma_W B(W \to \mu\nu_\mu)$, and $\sigma_W B(W \to \tau\nu_\tau)$. The ratios of these quantities are then used to test lepton universality. The formula used to calculate $\sigma_W B(W \to l\nu_l)$ is given in Appendix A.1.

D0, one of two experiments on the Tevatron collider, reports the following results for these quantities: $\sigma_W B(W \to e\nu_e) = 2.36 \pm 0.07 \pm 0.13$ nb, $\sigma_W B(W \to \mu\nu_\mu) = 2.09 \pm 0.23 \pm 0.11$ nb [10], and $\sigma_W B(W \to \tau\nu_\tau) = 2.38 \pm 0.13 \pm 0.13$ nb [12], where the first error is combined statistical and systematic uncertainty, and the last error is the uncertainty in the luminosity. The data were taken at a centre of mass energy of 1.8 TeV, and included 10338 $W \to e\nu_e$ candidates, 1665 $W \to \mu\nu_\mu$ candidates, and 1202 $W \to \tau\nu_\tau$ candidates. Using these results, we have computed the ratios g_μ/g_e and g_τ/g_μ

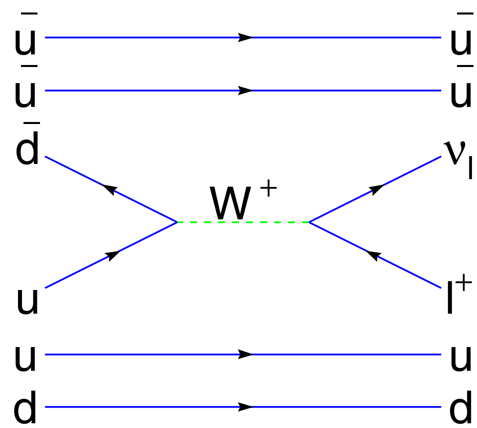

Figure 3: W production at $p\bar{p}$ colliders via a typical interaction.

and presented them in Table 4. The errors were computed under the assumption that the luminosity errors cancel in the ratio, and no other correlations exist. Also included in Table 4 is the ratio g_τ/g_e, which was calculated by Protopopescu [11] using the above D0 results.

The other Tevatron experiment is CDF, which has published the following results for the cross sections: $\sigma_W B(W \to e\nu_e) = 2.19 \pm 0.04 \pm 0.21$ nb and $\sigma_W B(W \to \mu\nu_\mu) = 2.21 \pm 0.07 \pm 0.21$ nb where the first error is statistical and the second systematic, including error in the luminosity, and $\sigma_W B(W \to \tau\nu_\tau) = 2.05 \pm 0.27$ nb where here the statistical and systematic errors have been combined in the calculation [13, 14, 15]. The data was taken at a centre of mass energy of 1.8 TeV, and included 2664 $W \to e\nu_e$ candidates, 1436 $W \to \mu\nu_\mu$ candidates, and 207 $W \to \tau\nu_\tau$ candidates. Using these results, they have determined the three coupling constant ratios as shown in Table 4.

5 The leptonic decays of the τ

An experimentally clean way to check lepton universality is through the leptonic decays of the τ, shown in Figures 4 (a) and (b), where the W is a virtual particle which propagates the force between the initial and final state fermions. In this case, the partial width is given by

$$\Gamma(\tau^- \to l^- \bar{\nu}_l \nu_\tau) = \left(\frac{g_\tau g_l}{8 m_W^2}\right)^2 \frac{m_\tau^5}{96\pi^3} f\left(\frac{m_l^2}{m_\tau^2}\right)(1+\delta_{RC}). \tag{4}$$

Equation 4 includes the mass corrections for the final state leptons, given by

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \tag{5}$$

The final term in Equation 4 is given by

$$(1+\delta_{RC}) = \left[1 + \frac{\alpha(m_\tau)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]\left[1 + \frac{3}{5}\frac{m_\tau^2}{m_W^2}\right] \tag{6}$$

where α is the electromagnetic coupling constant. The first term in square brackets in Equation 6 takes into account photon radiative corrections and the second term takes into account the non-local nature of the W propagator.

If we take the ratio of the partial widths of the two leptonic τ decays, the constants at the first vertex in Figure 4 (a) and (b) cancel, yielding the following relationship between g_μ and g_e,

$$\frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{g_\mu^2}{g_e^2}\left[\frac{f\left(\frac{m_\mu^2}{m_\tau^2}\right)}{f\left(\frac{m_e^2}{m_\tau^2}\right)}\right] \tag{7}$$

where $f(m_e^2/m_\tau^2) \approx 1$ and $f(m_\mu^2/m_\tau^2) = 0.9726$. Thus measurements of the two branching ratios, $B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$ and $B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ can be used to determine the ratio g_μ/g_e.

Several experiments have made precise measurements of these τ branching ratios using data from e^+e^- colliders, the most precise to date coming from the LEP and CESR colliders. The τ particles are produced in pairs via the interaction $e^+e^- \to \gamma/Z^0 \to \tau^+\tau^-$. Each τ particle subsequently decays in the detector.

Using the LEP data with centre of mass energy of approximately 91 GeV, it is possible to select the τ events and, knowing the total number of τ particles, the leptonic τ branching ratios can be measured directly using

$$B(\tau^- \to l^- \bar{\nu}_l \nu_\tau) = \frac{\text{Number of } \tau^- \to l^- \bar{\nu}_l \nu_\tau \text{ events}}{\text{Number of } \tau \text{ particles}} \tag{8}$$

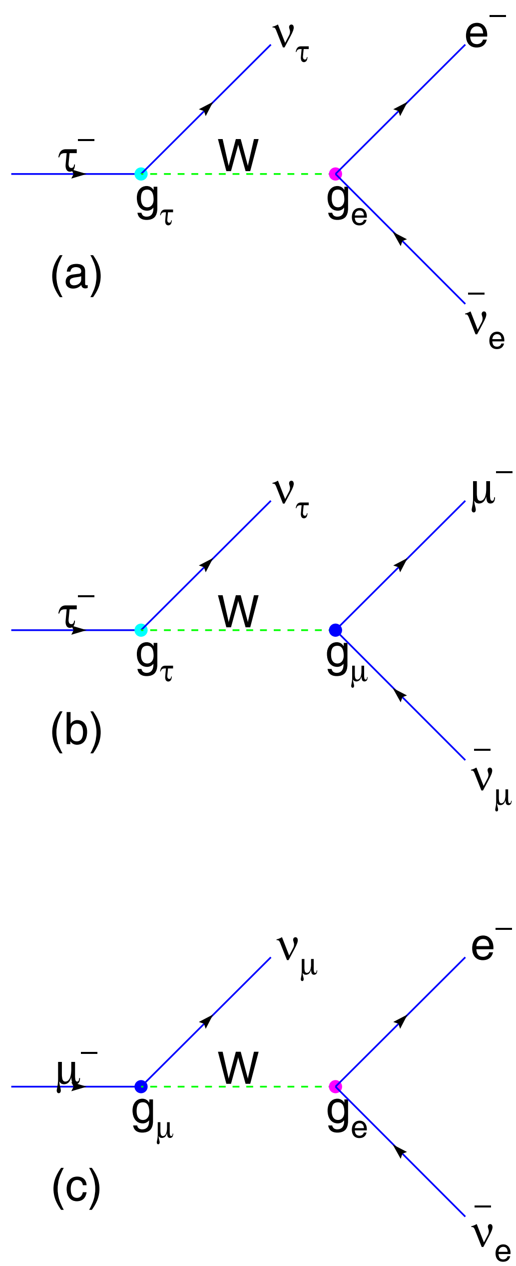

Figure 4: (a) $\tau^- \to e^- \bar{\nu_e} \nu_\tau$ decay, (b) $\tau^- \to \mu^- \bar{\nu_\mu} \nu_\tau$ decay, and (c) $\mu^- \to$ $\mathrm{e}^- \bar{\nu_{\mathrm{e}}} \nu_\mu$ decay.

with some corrections for efficiency and backgrounds. The procedure is explained in more detail in Appendix A.2. A typical number of τ candidates for an experiment using the entire LEP data set at the above energy is 186,000.

At CESR, selecting the τ events is more difficult. Hence, the branching ratios for the leptonic τ decays are determined by using the cross section and the measured luminosity to obtain the number of τ pairs, and by selecting specific τ pair event topologies (see Appendix A.2). In a recent analysis using data from the CLEO detector on CESR, the number of τ pairs was estimated to be $(3.250 \pm 0.046) \times 10^6$ τ pairs [16].

Results of this universality test using the latest measurements from the four LEP experiments and from CLEO were presented by Stugu [18] at the 1998 workshop on tau lepton physics. Since correlations are important in determining the uncertainties in the coupling constant ratio, Stugu has averaged the ratios g_μ/g_e when they are presented by the experiments. For the remaining experiments, he has computed averages of the branching ratios $B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$ and $B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$, and then used Equation 7 to determine g_μ/g_e. From these, he obtains the following overall average:

$$\frac{g_\mu}{g_e} = 1.0014 \pm 0.0024.$$

This result is more precise (by a factor of 10) than the results obtained by analyzing the decays of real W particles to leptons.

6 The electronic decay of τ and μ particles

One can also compare the decay $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ with the decay $\mu^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu$, shown in Figure 4 (a) and (c). In this case, it is the coupling constants at the last two vertices which will cancel in the ratio, allowing for a comparison of g_τ and g_μ. The ratio of the partial widths is given by

$$\frac{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}{\Gamma(\mu^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu)} = \frac{g_\tau^2 m_\tau^5 \ f\left(\frac{m_e^2}{m_\tau^2}\right)(1+\delta_{RC}^\tau)}{g_\mu^2 m_\mu^5 \ f\left(\frac{m_e^2}{m_\mu^2}\right)(1+\delta_{RC}^\mu)} \tag{9}$$

where $(1+\delta_{RC}^\tau)$ and $(1+\delta_{RC}^\mu)$ are the radiative corrections for the τ and μ decays, respectively. In terms of the branching ratios, this becomes

$$\frac{g_\tau^2}{g_\mu^2} = \frac{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}{B(\mu^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu)} \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{f\left(\frac{m_e^2}{m_\mu^2}\right)}{f\left(\frac{m_e^2}{m_\tau^2}\right)} \ \frac{(1+\delta_{RC}^\mu)}{(1+\delta_{RC}^\tau)} \tag{10}$$

where $f(m_e^2/m_\mu^2) = 0.9998$, and τ_μ and τ_τ are the lifetimes of the μ and the τ, respectively. The calculated numerical value for the ratio of the radiative corrections and mass corrections is [16]

$$\frac{f\left(\frac{m_e^2}{m_\mu^2}\right)}{f\left(\frac{m_e^2}{m_\tau^2}\right)} \frac{(1+\delta_{RC}^\mu)}{(1+\delta_{RC}^\tau)} = 0.9998. \tag{11}$$

Stugu [18] has used the world average lifetime, $\tau_\tau = 290.5 \pm 1.0$ fs, presented at the 1998 workshop on tau lepton physics, and the Particle Data Book 1998 [2] values for the other necessary quantities, to calculate the ratio

$$\frac{g_\tau}{g_\mu} = 1.0002 \pm 0.0025.$$

Equation 10 can be rearranged so that the τ lifetime is a function of the branching ratio $B(\tau^- \to e^-\bar{\nu_e}\nu_\tau)$. This relationship is plotted in Figure 5. The band is the Standard Model prediction under the assumption of lepton universality. Also shown is the OPAL measurement $B(\tau^- \to e^-\bar{\nu_e}\nu_\tau) = 17.81 \pm 0.09(\text{stat}) \pm 0.06(\text{syst})$ [17]. Figure 5 shows that this result is consistent with lepton universality.

In addition, it is possible to use the results from τ decay to calculate g_τ/g_e. This result is not independent of the other two results presented above, but is useful for comparison with results from other methods. By eliminating $B(\tau^- \to e^-\bar{\nu_e}\nu_\tau)$ between Equations 7 and 10, one arrives at the expression

$$\frac{g_\tau^2}{g_e^2} = B(\tau^- \to \mu^-\bar{\nu_\mu}\nu_\tau)\frac{m_\mu^5}{m_\tau^5}\frac{\tau_\mu}{\tau_\tau} \frac{f\left(\frac{m_e^2}{m_\mu^2}\right)}{f\left(\frac{m_\mu^2}{m_\tau^2}\right)} \frac{(1+\delta_{RC}^\mu)}{(1+\delta_{RC}^\tau)}\ , \tag{12}$$

and the corresponding result [18]

$$\frac{g_\tau}{ge} = 1.0013 \pm 0.0025.$$

7 The leptonic decays of pions

Before the discovery of τ particles or the production of real W particles, the decays of pions were already being used to test e/μ universality. Pions decay leptonically as shown in Figure 6. Although our understanding of each

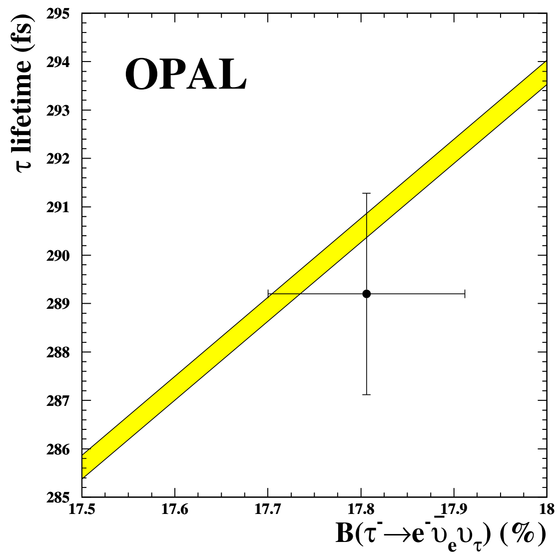

Figure 5: The OPAL τ lifetime is plotted against the $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ branching ratio. The band is the Standard Model prediction under the assumption of lepton universality, and its width represents the uncertainty due to the measured τ mass.

individual decay mode is hampered by QCD effects for which theoretical predictions become complicated, many of these effects will cancel if appropriate ratios are used. The π^+ is composed of u and $\bar{d}$ quarks, and decays weakly to either first or second generation leptons, as shown in Figure 6. The coupling at the first vertex (quarks to the W) is complicated by the hadronic contributions, which are the same in both Figure (a) and (b) and cancel in the ratio, allowing for comparison of the coupling strengths at the leptonic vertices. The blob at the first vertex signifies our ignorance of the hadronization process.

The pion decay width is given by

$$\Gamma(\pi \to l\nu_l) = \left(\frac{g_\pi g_l}{16 m_W^2}\right)^2 \frac{m_l^2 (m_\pi^2 - m_l^2)^2}{\pi m_\pi^3} \; V_{ud}^2 \, f_\pi^2 \, (1 + \delta_{RC}) \tag{13}$$

where the last term again represents radiative corrections which can be calculated for this interaction, f_π is a form factor which takes into account the

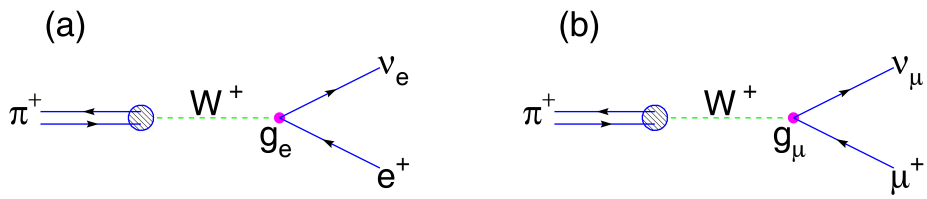

Figure 6: (a) $\pi^+ \to e^+\nu_e$ decay and (b) $\pi^+ \to \mu^+\nu_\mu$ decay.

hadronic contributions to the width, and V_{ud} is an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix that relates the weak eigenstates to the mass eigenstates. The ratio of the widths is thus

$$\frac{\Gamma(\pi \to e\nu_e)}{\Gamma(\pi \to \mu\nu_\mu)} = \left(\frac{g_e}{g_\mu}\right)^2 \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} \frac{(1+\delta_{RC}^e)}{(1+\delta_{RC}^\mu)} \tag{14}$$

where $(1+\delta_{RC}^e)/(1+\delta_{RC}^\mu) = 0.9626$ are the radiative corrections, including strong interaction effects. A complete description and detailed calculation is given by Decker and Finkemeier [22]. In practice, the ratio of the two decay widths is measured, and the experimentally determined ratio is then compared with a theoretical prediction of the ratio in order to determine $(g_e/g_\mu)^2$. The theoretical prediction is calculated using Equation 14, under the assumption of lepton universality ($g_e/g_\mu = 1$).

The two most recent experiments to measure these ratios have been done at TRIUMF and PSI [19, 20]. In both cases, a π^+ beam was produced, and the pions were stopped in a target where they decayed. Positrons from the decay $\pi \to e\nu_e$ were then detected, and the ratio $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu)$ was measured using the process summarized in Appendix A.3.

Using a data sample of $\approx 1.2 \times 10^5$ $\pi \to e\nu_e$ decays at TRIUMF, a value of $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu) = (1.2265 \pm 0.0034 \pm 0.0044) \times 10^{-4}$ was measured, where the first error is statistical and the second systematic. A preliminary calculation by Marciano and Sirlin (see [21] for the final calculation) of $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu) = (1.2345 \pm 0.001) \times 10^{-4}$ was used to determine the ratio

$$\frac{g_e}{g_\mu} = 0.9970 \pm 0.0023.$$

At PSI, a data sample of $\approx 3 \times 10^5$ $\pi \to e\nu_e$ decays from a total of 2.4×10^9 accumulated pions yielded the result $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu) = (1.2346 \pm 0.0035 \pm 0.0036) \times 10^{-4}$. The above result from Marciano and Sirlin was used to determine the ratio

$$\frac{g_e}{g_\mu} = 1.000 \pm 0.002.$$

In this work, we use a more recent theoretical calculation by Decker and Finkemeier [22], $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu) = (1.2356 \pm 0.0001) \times 10^{-4}$, and the averaged ratio of the decay widths from the 1998 Particle Data Book [2], $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu) = (1.230 \pm 0.004) \times 10^{-4}$, to calculate the following result:

$$\frac{g_e}{g_\mu} = 0.9977 \pm 0.0016.$$

Putting this in the same form as results from other universality tests, we find

$$\frac{g_\mu}{g_e} = 1.0023 \pm 0.0016.$$

The very small value of the ratio $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu)$ is surprising if one looks at phase space considerations only. The mass of the electron is very much less than that of a muon, hence it seems reasonable to expect the pion to decay preferentially to electrons. However, when one calculates the matrix element under an assumption of left-handed vector couplings to the W, the factor m_e^2/m_μ^2 enters into Equation 14. This ratio has historically provided one of the most important tests of the Standard Model predictions for the form of the couplings between elementary particles.

8 $\tau \to \mathrm{h}\nu_\tau$ decays and $\mathrm{h} \to \mu\nu_\mu$ decays

The ratio g_τ/g_μ can be determined by comparing the $\tau \to \mathrm{h}\nu_\tau$ decay with the $\mathrm{h} \to \mu\nu_\mu$ decay, where h stands for pions or kaons. The corresponding Feynman diagrams are shown in Figure 7. In (a), the τ is shown decaying to either a pion or a kaon. The π^+ (π^-) is composed of u and $\bar{d}$ ($\bar{u}$ and d) quarks; if the d quarks are replaced with s quarks, kaons are produced.

The corresponding decay widths are:

$$\Gamma(\tau \to \mathrm{h}\nu_\tau) = \left(\frac{g_\tau g_h}{16 m_W^2}\right)^2 \frac{(m_\tau^2 - m_h^2)^2}{2\pi m_\tau} V_{qq'}^2\, f_h^2\, (1 + \delta_{RC}^\tau), \qquad (15)$$

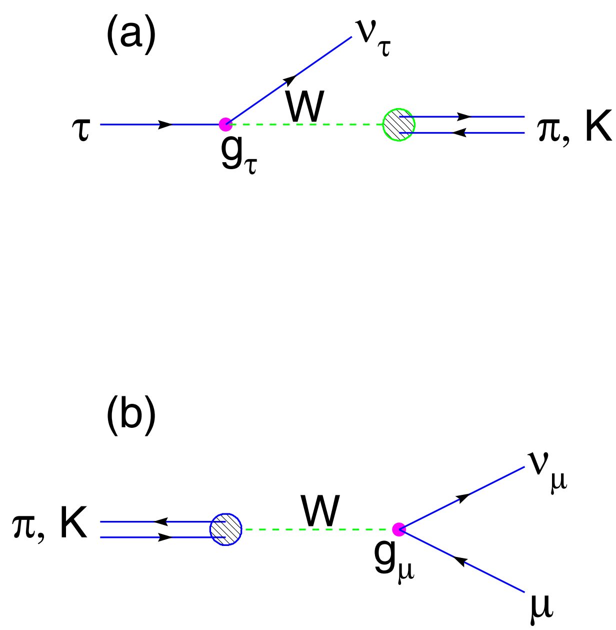

Figure 7: (a) $\tau^+ \to \mathrm{h}^+ \bar{\nu_\tau}$ decay, and (b) $\pi^+ \to \mu^+ \nu_\mu$ decay.

and

$$\Gamma(\mathrm{h} \to \mu\nu_\mu) = \left(\frac{g_h g_\mu}{16 m_W^2}\right)^2 \frac{m_\mu^2 (m_h^2 - m_\mu^2)^2}{\pi m_h^3} \; V_{qq'}^2 \, f_h^2 \, (1 + \delta_{RC}^h), \tag{16}$$

where $V_{qq'}$ is V_{ud} for h $= \pi$, and V_{us} for h $= K$. These expressions can be arranged in ratios to cancel out the problematic form factors at the hadronic vertices, yielding:

$$\frac{\Gamma(\tau \to \pi\nu_\tau)}{\Gamma(\pi \to \mu\nu_\mu)} = \left(\frac{g_\tau}{g_\mu}\right)^2 \frac{m_\tau^3}{2 m_\pi m_\mu^2} \left(\frac{1 - \frac{m_\pi^2}{m_\tau^2}}{1 - \frac{m_\mu^2}{m_\pi^2}}\right)^2 (1 + \delta_{\tau/\pi}) \tag{17}$$

and

$$\frac{\Gamma(\tau \to K\nu_\tau)}{\Gamma(K \to \mu\nu_\mu)} = \left(\frac{g_\tau}{g_\mu}\right)^2 \frac{m_\tau^3}{2 m_K m_\mu^2} \left(\frac{1 - \frac{m_K^2}{m_\tau^2}}{1 - \frac{m_\mu^2}{m_K^2}}\right)^2 (1 + \delta_{\tau/K}) \tag{18}$$

where $(1+\delta_{\tau/\pi})$ and $(1+\delta_{\tau/K})$ are the ratios of the radiative corrections. The radiative corrections in this case contain terms corresponding to radiative decays and strong interaction effects. A complete description and detailed calculation is given by Decker and Finkemeier [22], who present the following results:

$$\delta_{\tau/\pi} = (0.16^{+0.09}_{-0.14})\%$$

and

$$\delta_{\tau/K} = (0.90^{+0.17}_{-0.26})\%$$

In practice, it has not been possible until recently to adequately separate the $\tau \to \pi\nu_\tau$ decays from the $\tau \to K\nu_\tau$ decays, and so the sum of the measured branching ratios is used in the universality test. Expressed in terms of the widths, the sum is

$$B(\tau \to \pi\nu_\tau) + B(\tau \to K\nu_\tau) = \tau_\tau[\Gamma(\tau \to \pi\nu_\tau) + \Gamma(\tau \to K\nu_\tau)] \tag{19}$$

where τ_τ is the lifetime of the τ particle. Rearranging Equations 17 and 18, and using the relationship $B(\mathrm{h} \to \mu\nu_\mu) = \tau_h \Gamma(\mathrm{h} \to \mu\nu_\mu)$, Equation 19 can be written in the following form:

$$B(\tau \to \pi\nu_\tau) + B(\tau \to K\nu_\tau) = \left(\frac{g_\tau}{g_\mu}\right)^2 \frac{\tau_\tau m_\tau^3}{2 m_\mu^2} \{H_\pi + H_K\} \tag{20}$$

where

$$H_h = \frac{1+\delta_{\tau/h}}{m_h \tau_h}\left(\frac{1-\frac{m_h^2}{m_\tau^2}}{1-\frac{m_\mu^2}{m_h^2}}\right)^2 B(\mathrm{h}\to\mu\nu_\mu). \tag{21}$$

Equation 20 can be used to determine g_τ/g_μ.

The branching ratio $B(\tau^- \to \mathrm{h}^-\nu_\tau) \equiv B(\tau\to\pi\nu_\tau)+B(\tau\to K\nu_\tau)$ has been measured by CLEO and the four LEP experiments. At CLEO, $B(\tau^- \to \mathrm{h}^-\nu_\tau)$ was measured simultaneously with the values presented in Section 5 [16]; at the LEP experiments, the $\tau^- \to \mathrm{h}^-\nu_\tau$ branching ratio is measured directly as described in Section 5. (See also Appendix A.2.)

Stugu [18] has used the average branching ratio results presented by Heltsley [23] at the 1998 workshop on tau lepton physics, and the value for τ_τ from the same workshop, to calculate the ratio

$$\frac{g_\tau}{g_\mu} = 1.0037 \pm 0.0042.$$

9 Summary of experimental results

Table 5 presents the results of the universality tests of the preceding sections.

Method	g_μ/g_e	g_τ/g_e	g_τ/g_μ
$W\to l\nu_l$	0.97 ± 0.02	0.98 ± 0.02	1.00 ± 0.03
$\tau\to\mu\nu_\mu\nu_\tau/\tau\to e\nu_e\nu_\tau$	1.0014 ± 0.0024		
$\tau\to e\nu_e\nu_\tau/\mu\to e\nu_e\nu_\mu$			1.0002 ± 0.0025
$\tau\to\mu\nu_\mu\nu_\tau/\mu\to e\nu_e\nu_\mu$		1.0013 ± 0.0025	
$\pi\to e\nu_e/\pi\to\mu\nu_\mu$	1.0023 ± 0.0016		
$\tau\to \mathrm{h}\nu_\tau/h\to\mu\nu_\mu$			1.0037 ± 0.0042

Table 5: Results of the universality tests.

These data verify lepton universality at the level of 0.25% from τ decays, 0.16% from π decays, 0.42% from hadronic τ decays, and 2% from real W decays. Because of the spin 0 nature of pions and kaons, universality tests involving these hadrons (the last two methods in Table 5) test the coupling to a longitudinal W, and are hence slightly different from the tests involving leptonic τ decays or real W decays which probe the couplings of a transverse W.

10 Sensitivity to Higgs channels and new physics

The universality tests described above can be used to provide constraints on non-Standard Model phenomena. In this section, some background is provided on Standard Model couplings and on the Michel parameters. In particular, we discuss the sensitivity of the universality tests to the Michel parameter, η, which can be used to set limits on non-Standard Model couplings and on the mass of a charged Higgs particle. Next, we look at sensitivity to the radiative corrections and weak corrections in the leptonic decay widths, and sensitivity to the mass of the τ neutrino through phase space corrections. Lastly, we look at the importance of universality tests involving hadrons.

First, we shall consider the information obtained through τ leptonic decays. In the Standard Model, leptonic couplings to the W bosons are assumed to be of the form $V - A$. However, the most general form of the matrix element includes terms that correspond to all possible combinations of scalar, vector, and tensor couplings to charged leptons with left or right-handed chirality. These 10 terms have 10 associated coupling constants $g^{\lambda}_{\epsilon\omega}$, where λ is S, V, or T for scalar, vector or tensor couplings, and ϵ/ω are either L or R for the chirality of the initial/final state charged lepton. In the Standard Model, $g^{V}_{LL} = 1$ and all other $g^{\lambda}_{\epsilon\omega} = 0$.

The shapes of the τ decay spectra are more conveniently parameterized in terms of the four Michel parameters ρ, η, ξ, and δ where each parameter is a linear combination of the $g^{\lambda}_{\epsilon\omega}$. The Standard Model expectation values are $\rho = 3/4$, $\eta = 0$, $\xi = 1$, and $\xi\delta = 3/4$. Experimental values of these parameters have been obtained by fitting the $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ decay spectra. The Particle Data Group world averages from the latter mode of decay [2] are

$$\eta = -0.10 \pm 0.18$$

$$\rho = 0.741 \pm 0.030$$

$$\xi = 1.07 \pm 0.08$$

$$\xi\delta = 0.78 \pm 0.05.$$

The parameter η introduces corrections to the Standard Model leptonic decay widths of the τ, such that $\Gamma_l(\eta_l) = \Gamma^{(SM)}(1 + 4\eta_l \frac{m_l}{m_\tau} + ...)$. Thus, deviations from unity in the universality tests could be indicative of a non-zero η.

The η parameter is given by

$$\eta = \frac{1}{2} Re \left\{ g^V_{LL} g^{S*}_{RR} + g^V_{RR} g^{S*}_{LL} + g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR}) + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) \right\}. \quad (22)$$

A non-zero value of η would show that g^V_{LL} is not the only non-zero coupling constant in Equation 22. If one assumes g^V_{LL} to be dominant, and assumes all but the first term in Equation 22 to be zero, then it is possible to place constraints on g^S_{RR}, which corresponds to a Higgs-type coupling. Using the ratio $\mathrm{B}(\tau^- \to \mu^-\bar{\nu_\mu}\nu_\tau)/\mathrm{B}(\tau^- \to e^-\bar{\nu_e}\nu_\tau)$, Pich [24] obtains the value $\eta = 0.005 \pm 0.027$, which he then uses with the above assumptions to calculate the following constraint (90% C.L.):

$$-0.08 < \mathrm{Re}(g^S_{RR}) < 0.10.$$

Note that the width $\Gamma(\tau^- \to e^-\bar{\nu_e}\nu_\tau)$ is not sensitive to η because of the suppression factor m_e/m_τ.

The parameter η is related to g^S_{RR} through the relationship [25]

$$\eta = -\frac{g^S_{RR}/2}{1 + (g^S_{RR}/2)^2}, \quad (23)$$

which in turn is related to the Higgs mass through

$$g^S_{RR} = -m_l m_\tau \left(\frac{\tan\beta}{m_{H^\pm}}\right)^2, \quad (24)$$

where $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs fields in the Minimal Supersymmetric Model. Dova, Swain and Taylor [26] use a value of $\eta > -0.0186$ at the 95% C.L. which they obtained by a likelihood fit to the world averages for $\mathrm{B}(\tau^- \to \mu^-\bar{\nu_\mu}\nu_\tau)$ and $\mathrm{B}(\tau^- \to e^-\bar{\nu_e}\nu_\tau)$. They then use the above relationships to obtain the constraint $m_{H^\pm} > 1.86 \times \tan\beta$ GeV at the 95% C.L. This constraint is shown on the right hand side of Figure 8. It is complementary to the constraints from other analyses. For example, also shown are the constraints from direct searches at LEP which indicate $m_{H^\pm} > 54.5$ GeV [27], from measurements of the partial width $\Gamma(Z \to b\bar{b})$ which yield $\tan\beta > 0.7$ [28], from measurements of the branching

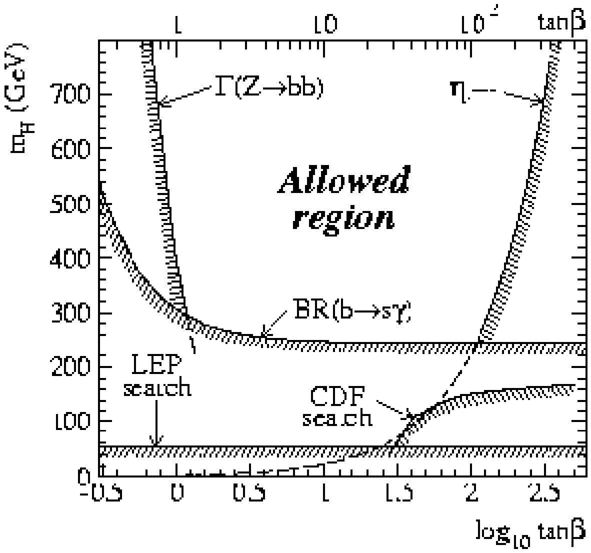

Figure 8: Constraints on $m_{H^\pm}$ as a function of $\tan\beta$ at the 95% C.L. from analysis of η and other analyses [26].

ratio B($b \to s\gamma$) which yield $m_{H^\pm} > 244 + 63/(\tan\beta)^{1.3}$ [29], and from the CDF search for charged Higgs bosons in the process $t \to bH^+$ which exclude regions of low $m_{H^\pm}$ and high $\tan\beta$ [30].

In addition, the universality test involving $\Gamma(\tau \to e\nu_e\nu_\tau)/\Gamma(\mu \to e\nu_e\nu_\mu)$ may become sensitive to the electromagnetic radiative corrections and the corrections to the W propagator, which contribute at the level of 0.14% [31], less than a factor of two below the current precision of the universality tests. As well, the phase-space correction would be influenced by a non-zero ν_τ mass. With the current best limit of $m_{\nu_\tau} < 18.2$ MeV [2], this effect is at the level of 0.08%, indicating that future improvements in the precision of universality tests may provide both a means of placing constraints on m_{ν_τ}, and a check on electroweak and radiative corrections.

The information provided by universality tests involving pions and kaons is complementary to that provided by tests involving only leptons. For example, as noted above, the value of g_μ/g_e obtained from tests involving the fully leptonic decay modes of the τ would be affected (through η) by the scalar coupling of a charged Higgs particle, whereas, the value of g_μ/g_e obtained from tests involving decays of pions would be unaffected.

The value of g_τ/g_μ obtained through the comparison of $\Gamma(\tau \to \mathrm{h}\nu_\tau)$ to $\Gamma(h \to \mu\nu_\mu)$ could unveil the presence of scalar-exchange contributions with couplings proportional to some power of the charged-lepton mass. Such couplings are hypothesized in R-parity violating extensions of supersymmetric models, and universality results have been used to set limits on parameters within these models [32].

All of these phenomena could potentially cause the values measured in lepton universality tests to deviate from unity. However, in each case, the deviation is caused by effects not actually associated with the coupling constants g_τ, g_μ, and g_e. If a deviation from unity were to be observed, it would be necessary to rule out the above contributions before one could assume that the coupling constants themselves differ. A scenario where $g_\tau \neq g_\mu \neq g_e$ would indicate that the lepton families differ in more than mass and lepton number.

11 Conclusions

Tests of lepton universality allow us to investigate whether mass and lepton number are the only differences between the three lepton families, as well as allowing us to search for other types of new physics. Lepton universality has been tested with a precision of approximately 0.25% using τ and μ leptonic decays, 0.42% using τ hadronic decays, and 0.16% using leptonic π decays. To date, this has not provided evidence of non-Standard Model physics. At present, the results from the decays of real W particles are approximately a factor of 10 less precise than those listed above; however, larger data sets in this area may provide more precise tests in the future, especially when data become available from Tevatron II, the LHC, and possibly a new linear collider.

A Branching Ratio Information

A.1 W decays

The expected number of WW pairs produced in the OPAL detector is given by

$$N_{WW} = \sigma_{WW}\,\mathcal{L} \tag{25}$$

where σ_{WW} is the cross section for the interaction $e^+e^- \rightarrow WW$ and $\mathcal{L}$ is the luminosity. Each W may decay to one of three leptonic states ($e\nu_e$, $\mu\nu_\mu$, $\tau\nu_\tau$), or to a hadronic state ($q\bar{q}$). In an event with two W particles, there are 10 possible event topologies, six of which are purely leptonic. The expected number of events in the i^{th} W pair topology is given by

$$N_i = \sigma_{WW}\,\mathcal{L}\,B_1 B_2 \tag{26}$$

where B_1 and B_2 are the branching ratios for the decay modes of the two W particles. The number of events within the i^{th} WW topology that are actually observed in the detector, N_i^{obs}, is affected by the efficiency of the selection, ϵ_i, and by the fractional background in the sample, f_i^{bg}, so that we have

$$N_i = \frac{N_i^{obs}(1 - f_i^{bg})}{\epsilon_i} = \sigma_{WW}\,\mathcal{L}\,B_1 B_2. \tag{27}$$

The efficiency and background are determined by Monte Carlo simulations. One can then predict an expected number of observed events, μ_i, where

$$\mu_i = \sigma_{WW}\,\mathcal{L}\sum_j \epsilon_{ij} B_j, \tag{28}$$

ϵ_{ij} is the efficiency for selecting the j^{th} mode of W decay within the i^{th} W pair topology, and B_j is the branching ratio for the j^{th} mode of W decay.

One can use the observed number of events in each of the 10 topologies in a likelihood fit using a Poisson probability distribution. The branching ratios $B(W \rightarrow e\nu_e)$, $B(W \rightarrow \mu\nu_\mu)$, $B(W \rightarrow \tau\nu_\tau)$, and the cross section σ_{WW} are input parameters which determine μ_i. These four parameters are solved for in the fit, under the assumption that $B(W \rightarrow e\nu_e) + B(W \rightarrow \mu\nu_\mu) + B(W \rightarrow \tau\nu_\tau) + B(W \rightarrow q\bar{q}) = 1$. A second likelihood fit is used to extract the branching ratio $B(W \rightarrow q\bar{q})$ where, in addition to the unitarity assumption of the first fit, lepton universality is also assumed [33].

At the Tevatron collider, the W particles are produced singly. The quantity that is measured is the product of the branching ratio and the cross section, and is determined using the relationship

$$\sigma_W B(W \rightarrow l\nu_l) = \frac{N_l^{obs} - N_l^{bg}}{\epsilon\,\mathcal{L}}, \tag{29}$$

where σ_W is the cross section for W production, N_l^{obs} is the number of $W \rightarrow l\nu_l$ candidates observed in the detector, N_l^{bg} is the number of predicted background events in the $W \rightarrow l\nu_l$ sample, ϵ is the predicted efficiency of the selection, and $\mathcal{L}$ is the luminosity.

A.2 τ decays

At LEP, until the end of 1995, τ particles were produced via the interaction $e^+e^- \to Z^0 \to \tau^+\tau^-$. However, the Z^0 could also decay to other lepton families, or to quarks which subsequently hadronize. The $\tau^+\tau^-$ events are separated from the other LEP events by requiring that the event have two back to back jets, as well as low track and cluster multiplicities. Other criteria concerning energy deposition, and signals in the muon chambers, are also applied. The τ selection yields a sample of τ particles with approximately 2% background.

After the τ pairs are selected, they are used in further analyses to identify specific modes of τ decay. The branching ratio for the process $\tau \to l\nu_l\nu_\tau$, for example, is then given by

$$B(\tau \to l\nu_l\nu_\tau) = \frac{N_l^{obs}(1 - f^{bg})}{N_\tau^{obs}(1 - f^{non-\tau})\,\epsilon} \tag{30}$$

where N_l^{obs} is the number of decays of the type $\tau \to l\nu_l\nu_\tau$ observed in the detector, f^{bg} is the fractional number of background events contaminating the $\tau \to l\nu_l\nu_\tau$ sample, N_τ^{obs} is the number of τ candidates selected by the τ selection, $f^{non-\tau}$ is the fractional background in the τ sample, and ϵ is the efficiency for selecting $\tau \to l\nu_l\nu_\tau$ events.

At CESR, it is more difficult to apply a τ selection to the data. Because the CESR ring is operated at a lower centre of mass energy than LEP, multihadronic events ($e^+e^- \to q\overline{q}$) have much lower multiplicities and cannot be separated from the τ pair sample. Hence, the $\tau^+\tau^-$ event topologies are selected instead, in a manner similar to the analysis of W pairs at LEP2. Various methods have been used to determine the branching ratios of τ decay. In the CLEO determination of the τ branching ratios presented here, nine τ pair decay topologies are selected: ee, $\mu\mu$, hh, $e\mu$, eh, μh, ρe, $\rho\mu$, and ρh, where h is a charged pion or kaon, and ρ signifies an h accompanied by at least one π^0 [16].

Products of the branching ratios for the τ pair decays to the i^{th} topology are computed as

$$B_1\,B_2 = \frac{N_i^{obs}(1 - f_i^{bg})}{\epsilon_i\,N_{\tau\tau}\,(2 - \delta_{12})} \tag{31}$$

where B_1 and B_2 are the branching ratios for the decay modes of the two τ particles, N_i^{obs} is the number of events within the i^{th} $\tau\tau$ topology that are observed in the detector, f_i^{bg} is the fractional background in the sample, ϵ_i is the efficiency of the selection, $N_{\tau\tau}$ is the total number of τ pairs, and the Kronecker-δ accounts for the case when $1 = 2$ (i.e. the ee topology for example).

The number of τ pairs is calculated using

$$N_{\tau\tau} = \sigma_0 \left(1 + \delta_\tau\right) \mathcal{L} \tag{32}$$

where σ_0 is the point cross section, $(1 + \delta_\tau)$ is a correction to the point cross section for nonzero τ mass effects and radiative corrections, and $\mathcal{L}$ is the luminosity.

The measured product branching ratios are combined to yield the values of $B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$, $B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$, $B(\tau^- \to \mathrm{h}^-\nu_\tau)$, and the ratios $B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)/B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$ and $B(\tau^- \to \mathrm{h}^-\nu_\tau)/B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$. These five results are calculated using five separate χ^2 fits, each with six independent combinations of measured product branching ratios. Hence, in each fit, there are six input values, one free parameter which is to be determined, and five degrees of freedom.

A.3 Pion decays

Measurement of the branching ratio $\mathrm{B}(\pi \to e\nu_e)$ involves separating the signal events from an overwhelmingly large background (the decay $\pi \to \mu\nu_\mu$ followed by the decay $\mu \to e\nu_e\nu_\mu$). At both TRIUMF and PSI, this measurement involves stopping pions in a target where they subsequently decay. The positrons produced in both of the above decays exit the target and are then detected. There are two main observable properties used to differentiate between these types of events: the time, t, between the pion stopping in the target and the arrival of the positron at the detector, and the energy of the positron. The pion lifetime is 2.6033×10^{-8} s while that of the muon is 2.19703×10^{-6} s, hence positrons arriving as a result of the $\pi - \mu - e$ chain will arrive later. Also, since the muon usually deposits all of its kinetic energy in the stopping target before decaying, the positron from the $\pi - \mu - e$ chain has less energy.

At PSI, the total number of stopped pions is counted, as well as the number of $\pi \to e\nu_e$ decays [20]. The ratio $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu)$ is determined directly from the number of events of each type, and then corrected for backgrounds and efficiency.

At TRIUMF, the total number of positrons is counted, and then the time spectrum (the number of positrons versus t) is fitted to a theoretical model in order to determine the raw branching ratio [19]. The theoretical model includes the time spectrum produced by the signal events ($\pi \to e\nu_e$), those produced by the $\pi-\mu-e$ chain, and a component for background muons. The fit is performed both in the high energy region which is dominated by signal, and the low energy region which is dominated by the $\pi - \mu - e$ background. In the standard fitting procedure, the total number of positrons, the raw branching ratio, the pion stopping time, and the fractional background are all free parameters. The raw branching ratio is then corrected for backgrounds and efficiency.

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